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Solving Quadratic Fractional Programming Problem via Feasible Direction Development and Modified Simplex Method

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Abstract:

In this paper, I have expanded, the given algorithms [11,12] in to Quadratic Fractional Programming Problem (QFPP), I have used two methods to solve the problem one of them is Modified Simplex Method and the other Feasible Direction Development. And, then a good results I have been obtained as compared the ways of solution of the two methods. The computer application of our algorithms has also been discussed by solving constructed numerical example using Matlab2011 version 7.12.0.635 (R2011a). Finally this paper ends up with some conclusions.

Keywords: QFPP, Modified Simplex Method, Feasible Direction Development.

1. Introduction

Quadratic fraction problems (i.e. ratio of objectives that have numerator and denominator) have attracted considerable research and interest since they are useful in production planning, financial and corporative planning, health care and hospital planning. Several methods to solve such problems are proposed in (1962) [4], their method depends on transforming this Linear Fractional Programming Problem (LFPP) to an equivalent Linear Program. Sing (1981) [9] did a useful study about the optimality condition in Fractional Programming. Archana Khurana and Arora (2011) studied for solving a QFP when some of its constraints are homogeneous [1]. Salih(2010) [10] studied and suggested Feasible Direction and modified Simplex method to solve LFPP after comparison them. Abdulrahim (2011) [2] studied and suggested Wolf's method and Modified Simplex method to solve QPP after comparison them.

In (1962) Chames and Coopr [4] showed that by a simple transformation the original LFP problem can be reduced to an LPP that can therefore be solved using a regular Simplex method for LP. It was found that this approach is very useful for mathematicians because most theoretical results developed in LP could be relatively easily expanded to include LFPP [3]. Enkhbat, Bazrsadand Enkhbatyan (2011) a method for Fractional Programming [5]. Fang, Gao, Sheu and Xing (2009) global optimization for a class of Fractional Programming Problems[6]. Jeflea (2003) a parametric studies for solving Nonlinear Fractional Problems [7]. Fukushima and Hayashi (2008) Quadratic Fractional Programming Problems with Quadratic constraints [8].

In order to extend this work have studied and developed a Feasible Direction Method to solve LFPP which is defined by Tantawy [12,10], and developed a Feasible direction method to solve QFPP have suggested an approach to solve the same problem by using the Modified Simplex Method.

Finally we have show some numerical results and comparisons between the above techniques.

2.Feasible Direction Development Method

The mathematical programming problem for QFPP can be formulated as follows:

$$\text{Maximize(Minimize) } Z = \frac{(c^t x + \gamma)(e^t x + \delta)}{(d^t x + \beta)(f^t x + \varepsilon)} \qquad (2.1)$$

Subject to:

$$x \in X = \{x;\ AX \leq b\}$$

Tantawy reformulated the constraints as follows:

$$a^t x \leq b_i \quad, i = 1,2,3, \dots, m+n \qquad (2.2)$$

Where $x \in R^n$, A is an $(m+n) \times n$ matrix; c, e, d and f are $n-$vectors; $b \in R^{m+n}$ and$\gamma, \beta, \delta, \varepsilon$ are scalar constants. Moreover$f^t x + \varepsilon$, $d^t x + \beta > 0$ every where in X, and a_i^t represents the $i-$th row of the given matrix A, the method start with an initial feasible point then a sequence of feasible directions toward optimal extreme point of this problem.

In general if x_{k-1} is considered as a feasible point that obtained at iteration $k-1$, where $k = 1,2, \dots$ then at iteration k the procedure finds a new feasible point x_k that is given by

$$x_k = x_{k-1} + \alpha_{k-1} d_{k-1} \qquad (2.3)$$

Where $d_{k-1} = H_{k-1} \theta_{k-1}$ (2.4)

Here H_{k-1} is an $(n \times n)$ symmetric matrix given by

$$H_{k-1} = \begin{cases} I & for\ \ k = 1 \\ H_{k-1}^q & for\ \ k > 1 \end{cases} \qquad (2.5)$$

Also H_{k-1}^q for each active constraint $s, s = 1,2, \dots q$defined as follow

$$H_{k-1}^q = H_{k-1}^{s-1} - \frac{H_{k-1}^{s-1} a_s\, a_s^t H_{k-1}^{s-1}}{a_s^t H_{k-1}^{s-1} a_s} \qquad (2.6)$$

and

$$\alpha_{k-1} = \min\{g_i / g_i = \frac{b_i - a_i^t x_{k-1}}{a_i^t d_{k-1}}, \text{and} g_i > 0\}, i = 1,2, \dots, m+n \qquad (2.7)$$

It is clear from the relation that α_{k-1} is always positive. And θ_{k-1} is computed at the given point x_{k-1} to represent the local gradient at this point and defined as:

$$\theta_{k-1} = (d^t x_{k-1} + \beta)(f^t x_{k-1} + \varepsilon)ce - (c^t x_{k-1} + \gamma)(e^t x_{k-1} + \delta)df \qquad (2.8)$$

Then consider a linear programming

$$\text{Maximize } Z^* = \theta_k^t x \qquad (2.9)$$

Subject to: $x \in X = \{x; AX \leq b\}$

With $\theta_k = (d^t x_k + \beta)(f^t x_k + \varepsilon)ce - (c^t x_k + \gamma)(e^t x_k + \delta)df$

In this method the point x_k will be optimal solution of the linear programming defined by (2.9), if there exist u ≥ 0 such that:

$$A_r^t u = \theta_k \text{ or } u = (A_r A^t)^{-1} A_r \theta_k \qquad (2.10)$$

When A_r is a sub-matrix of the given matrix A containing only the coefficients of the set of active constraints at the current point x_k. For more details see [12].

3. Algorithm for Feasible Direction Development Method

An algorithm for solve QFPP by feasible direction method which can be summarized as follows [12]:

Step1: Set $H_0 = I, d_0 = \theta_0$, let x_0 be an initial feasible point and use relation (2.7) to compute α_0.

Step2: Apply relation (2.3) to find a new solution x_k.

Step3: Apply relation (2.10) to compute u, if$u \geq 0$ stop. The current solution x_k is the optimal solution otherwise go to step 4.

Step4: Set $k = k + 1$, apply relations (2.6), (2.4) and (2.7) to compute H_{k-1}, d_{k-1} and α_{k-1} respectively and go to step 2.

4. Numerical Example

We consider the following QFPP as:

$$\text{Maximize } Z = \frac{(x_2+1)(x_1+x_2+3)}{(x_1+4)(x_1+x_2+2)} \qquad \text{Subject to: } \begin{array}{r} -x_1 + x_2 \leq 1 \\ x_2 \leq 2 \\ x_1 + 2x_2 \leq 7 \\ x_1 \leq 5 \\ x_1, x_2 \geq 0 \end{array}$$

Solution: Solving the example by using Feasible Direction Method as follows:

Step1: $k = 1, d_0 = \theta_0, H_0 = I, c^t = (0 \quad 1), d^t = (1 \quad 0), e^t = (1 \quad 1),\ f^t = (1 \quad 1), \gamma = 1, \delta = 3, \beta = 4, \varepsilon = 2$

$$A = \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 2 \\ 1 & 0 \end{pmatrix} \text{ Let } x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ be an initial feasible point}$$

$$\theta_0 = \theta_{1-1} = (d^t x_0 + \beta)(f^t x_0 + \varepsilon)ce - (c^t x_0 + \gamma)(e^t x_0 + \delta)df$$

$$= \left[(1 \quad 0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 4\right]\left[(1 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2\right]\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \left[(0 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1\right]\left[(1 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \left[(1+4)(2+2)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right] - \left[(1+1)(2+3)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 0 \\ 20 \end{pmatrix} - \begin{pmatrix} 10 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 20 \end{pmatrix} = d_0$$

$$\alpha_0 = \min\{g_i; g_i = (b_i - a_i^t x_0)/a_i^t d_0\} = \min\{1/30, 1/20, 2/15\} = 1/30$$

$$g_1 = (b_1 - a_1^t x_0)/a_1^t d_0 = \left[1 - (-1 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]/(-1 \quad 1)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = 1/30$$

$$g_2 = (b_2 - a_2^t x_0)/a_2^t d_0 = \left[2 - (0 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]/(0 \quad 1)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = 1/20$$

$$g_3 = (b_3 - a_3^t x_0)/a_3^t d_0 = \left[7 - (1 \quad 2)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]/(1 \quad 2)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = 2/15$$

$$g_4 = (b_4 - a_4^t x_0)/a_4^t d_0 = \left[5 - (0 \quad 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]/(0 \quad 1)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = -1/5$$

Step2: $x_k = x_{k-1} + \alpha_{k-1} d_{k-1}$

$$x_1 = x_0 + \alpha_0 d_0 = \begin{pmatrix}1\\1\end{pmatrix} + (1/30)\begin{pmatrix}-10\\20\end{pmatrix} = \begin{pmatrix}2/3\\5/3\end{pmatrix}$$

Step3: $A_r = (-1 \quad 1)$, $r = 1$, find u then x_1 is not optimal solution.

Step4: $k = 1 + 1 = 2$

$$H_0 = I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$H_1 = H_0 - (H_0 a_1\, a_1^t H_0 / a_1^t H_0 a_1)$$

$$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \left[\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}(-1 \quad 1)\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} / (-1 \quad 1)\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}\right]$$

$$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \begin{pmatrix}1/2 & -1/2\\-1/2 & 1/2\end{pmatrix} = \begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$$

$$\theta_1 = \theta_{2-1} = (d^t x_1 + \beta)(f^t x_1 + \varepsilon)ce - (c^t x_1 + \gamma)(e^t x_1 + \delta)df$$

$$= \left[(1 \quad 0)\begin{pmatrix}2/3\\5/3\end{pmatrix} + 4\right]\left[(1 \quad 1)\begin{pmatrix}2/3\\5/3\end{pmatrix} + 2\right]\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} -$$

$$\left[(0 \quad 1)\begin{pmatrix}2/3\\5/3\end{pmatrix} + 1\right]\left[(1 \quad 1)\begin{pmatrix}2/3\\5/3\end{pmatrix} + 3\right]\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \begin{pmatrix}-128/9\\182/9\end{pmatrix}$$

$$d_1 = H_1 \theta_1 = \begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}\begin{pmatrix}-128/9\\182/9\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$$

We find α_1,$\alpha_1 = \min\{1/9, 1/3, 13/9\} = 1/9$

$$g_1 = (b_1 - a_1^t x_1)/a_1^t d_1 = \left[1 - (-1 \quad 1)\begin{pmatrix}2/3\\5/3\end{pmatrix}\right] \Big/ (-1 \quad 1)\begin{pmatrix}3\\3\end{pmatrix} = \frac{0}{0}$$

$$g_2 = (b_2 - a_2^t x_1)/a_2^t d_1 = \left[2 - (0 \quad 1)\begin{pmatrix}2/3\\5/3\end{pmatrix}\right] \Big/ (0 \quad 1)\begin{pmatrix}3\\3\end{pmatrix} = 1/9$$

$$g_3 = (b_3 - a_3^t x_1)/a_3^t d_1 = \left[7 - (1 \quad 2)\begin{pmatrix}2/3\\5/3\end{pmatrix}\right] \Big/ (1 \quad 2)\begin{pmatrix}3\\3\end{pmatrix} = 1/3$$

$$g_4 = (b_4 - a_4^t x_1)/a_4^t d_1 = \left[5 - (1 \quad 0)\begin{pmatrix}2/3\\5/3\end{pmatrix}\right] \Big/ (1 \quad 0)\begin{pmatrix}3\\3\end{pmatrix} = 13/9$$

Now we find x_2 as follows: $x_2 = x_1 + \alpha_1 d_1 = \begin{pmatrix}2/3\\5/3\end{pmatrix} + (1/9)\begin{pmatrix}3\\3\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$ We test if x_2 is an optimal point$A_r^t u = \theta_k$ First we have to find θ_2

$$\theta_2 = \theta_{3-2} = (d^t x_2 + \beta)(f^t x_2 + \varepsilon)ce - (c^t x_2 + \gamma)(e^t x_2 + \delta)df$$

$$= \left[(1 \quad 0)\begin{pmatrix}1\\2\end{pmatrix} + 4\right]\left[(1 \quad 1)\begin{pmatrix}1\\2\end{pmatrix} + 2\right]\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} -$$

$$\left[(0 \quad 1)\begin{pmatrix}1\\2\end{pmatrix} + 1\right]\left[(1 \quad 1)\begin{pmatrix}1\\2\end{pmatrix} + 3\right]\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-18\\25\end{pmatrix}$$

$$A_2^t = \begin{pmatrix}-1 & 0\\1 & 1\end{pmatrix}, \quad u = \begin{pmatrix}u_1\\u_2\end{pmatrix} \to A_2^t u = \theta_2 \to \begin{pmatrix}-1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}-18\\25\end{pmatrix}$$

We get the following system $\begin{matrix}-u_1 & = -18\\ u_1 + u_2 & = 25\end{matrix}$ After solve the system, we get$u_1 = 18$ and $u_2 = 7$ Since $u = \begin{pmatrix}u_1\\u_2\end{pmatrix}$ and $u_i \geq 0$ then x_2 is an optimal solution $x_2 = \begin{pmatrix}1\\2\end{pmatrix}$ is an optimal point and $Max.Z = 18/25$

5.Modified Simplex Method Development:

Simplex method is developed by Dantzig in (1947). The Simplex method provides a systematic algorithm which consists of moving from one basic feasible solution (on vertex) to another in prescribed manner such that the value of the objective function is improved. This procedure of jumping from vertex to vertex is repeated. If the objective function is improved at each jump, then no basis can ever be repeated and there is no need go to back to vertex already covered. Since the number of vertices is finite, the process must lead to the optimal vertex in a finite number of steps.

The Simplex algorithm is an iterative (step by step) procedure for solving linear programming problems. It consists of:

i) Having a trail basic feasible solution to constraint equations.

ii) Testing whether is an optimal solution.

iii) Improving the first trial solution by a set of rules, and repeating the process till an optimal solution is obtained.

For more details [11].

6. Algorithm for Modified Simplex Method

An algorithm for solve QFPP by Modified Simplex Method which can be summarized as follows [11]:

Step1: Write the standard form of the problem, by introducing slack and artificial variables to constraints, and write starting Simplex table.

Step2: Calculate the Δ_j by the following formula $\Delta_j = Z_2\Delta_{j1} - Z_1\Delta_{j2}$, then write it in the starting Simplex table.

Step3: Find the solution by using Simplex process.

Step4: Check the solution for feasibility in step3, if it is feasible then go to step5, otherwise use dual Simplex method to remove infeasibility.

Step5: Check solution for optimality if all$\Delta_j \geq 0$, then the solution is optimal, otherwise go to step3.

Using Modified Simplex Method to solve the numerical example to apply simplex process, first we find Δ_{j1} and Δ_{j2} from the coefficient of numerator and denominator of objective function respectively, by using the following formula:

$$\Delta_{ji} = c^*_{Bi}x_{ji} - c^*_{ji} \quad , i = 1,2, j = 1,2,\dots,m+n$$

$$z_1 = c_{B1}v_B + \gamma,\ z_2 = c_{B2}v_B + \delta,\ Z_3 = c_{B3}v_B + \beta,\ Z_4 = c_{B4}v_B + \varepsilon$$

$$\gamma = 1, \delta = 3, \beta = 4, \varepsilon = 2$$

$$Z_1 = z_1z_2, Z_2 = z_3z_4, Z_1 = c^*_{B1}v_B + \gamma\delta, Z_2 = c^*_{B2}v_B + \beta\varepsilon$$

$$c^*_{B1} = c_{B1}c_{B2}, c^*_{B2} = c_{B3}c_{B4}$$

$$c^*_{j1} = c_{j1}c_{j2},\ c^*_{j2} = c_{j3}c_{j4}$$

$$Z = Z_1/Z_2$$

In this approach we define the formula to find Δ_j from Z_1, Z_2, Δ_{j1} and Δ_{j2} as follows:

$$\Delta_j = Z_2\Delta_{j1} - Z_1\Delta_{j2}$$

Here $c_{ji}^*, c_{j1}, c_{j2}, c_{j3}, c_{j4}$ are the coefficients of the basic and non-basic variables in the objective function and $c_{Bi}^*, c_{B1}, c_{B2}, c_{B3}, c_{B4}$ are the coefficients of the basic variables in the objective function, $j = 1,2,\dots,m+n, i = 1,2$

The first table in Modified Simplex Method is follows:

					c_{j1}	0	1	0	0	0	0	
					c_{j2}	1	1	0	0	0	0	
					c_{j3}	1	0	0	0	0	0	
					c_{j4}	1	1	0	0	0	0	
B.V.	c_{B1}	c_{B2}	c_{B3}	c_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_3	0	0	0	0	1	-1	①	1	0	0	0	$1/1 = 1$
x_4	0	0	0	0	2	0	1	0	1	0	0	$2/1 = 2$
x_5	0	0	0	0	7	1	2	0	0	1	0	$7/2 = 3.5$
x_6	0	0	0	0	5	1	0	0	0	0	1	–
$Z_1 = 3$					Δ_{j1}	0	-1	0	0	0	0	
$Z_2 = 8$					Δ_{j2}	-1	0	0	0	0	0	
$Z = Z_1/Z_2 = 3/8$					Δ_j	3	$\boxed{-8}$	0	0	0	0	

			c_{j1}^*	0	1	0	0	0	0	
			c_{j2}^*	1	0	0	0	0	0	
B.V.	c_{B1}^*	c_{B2}^*	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_3	0	0	1	-1	①	1	0	0	0	$1/1 = 1$
x_4	0	0	2	0	1	0	1	0	0	$2/1 = 2$
x_5	0	0	7	1	2	0	0	1	0	$7/2 = 3.5$
x_6	0	0	5	1	0	0	0	0	1	–
$Z_1 = 3$			Δ_{j1}	0	-1	0	0	0	0	
$Z_2 = 8$			Δ_{j2}	-1	0	0	0	0	0	
$Z = Z_1/Z_2 = 3/8$			Δ_j	3	$\boxed{-8}$	0	0	0	0	

In the above table the feasible solution is $x_1 = 0, x_2 = 0, x_3 = 1,\ x_4 = 2\ ,\ x_5 = 7,$ $x_6 = 5$ and $Max.Z\ = 3/8$ this solution is not optimal, because all Δ_j not greater than or equal 0, then we find next feasible solution, as follows:

Since $\Delta_j = -8$, then we select x_2 to be entering vector and we find minimum ratio where min ratio = min $\{\frac{V_B}{x_2}; x_i > 0$ and min{1,2,7/2} =1 in the first row will be x_3 the outing vector.

Pivot element is at row 1 column 2

By applying Simplex technique we get:

			c_{j1}^*	0	1	0	0	0	0	
			c_{j2}^*	1	0	0	0	0	0	
B.V.	c_{B1}^*	c_{B2}^*	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_2	1	0	1	-1	1	1	0	0	0	—
x_4	0	0	1	①	0	-1	1	0	0	$1/1 = 1$
x_5	0	0	5	3	0	-2	0	1	0	$5/3 = 1.6$
x_6	0	0	5	1	0	0	0	0	1	$5/1 = 5$
$Z_1 = 8$			Δ_{j1}	-1	0	1	0	0	0	
$Z_2 = 12$			Δ_{j2}	-1	0	0	0	0	0	
$Z = 8/12$			Δ_j	$\boxed{-4}$	0	12	0	0	0	

For testing optimality solution must be all $\Delta_j \geq 0$ but here all Δ_j not greater than zero, and then the solution is not optimal. Repeat the same approach to find next feasible solution, and get:

			c_{j1}^*	0	1	0	0	0	0	
			c_{j2}^*	1	0	0	0	0	0	
B.V.	c_{B1}^*	c_{B2}^*	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_2	1	0	2	0	1	0	1	0	0	
x_1	0	1	1	1	0	-1	1	0	0	
x_5	0	0	2	0	0	1	-3	1	0	
x_6	0	0	4	0	0	1	-1	0	1	
$Z_1 = 18$			Δ_{j1}	0	0	0	1	0	0	
$Z_2 = 25$			Δ_{j2}	0	0	-1	1	0	0	
$Z_3 = Z_1/Z_2 = 18/25$			Δ_j	0	0	18	7	0	0	

Then the solution is $x_1 = 1,\ x_2 = 2$ and $Max.Z\ =\ 18/25$Since all $\Delta_j \geq 0$ then the solution is optimal. **Note:** The optimal solution is the same as used Tantawy approach.

7. Discussion

The conclusion is that the Modified Simplex Method is better than Feasible Direction Development Method, but I don't agree with that, because simplex Method deals with variables not more than $n = 10$ and then we go to Revised Simplex Method, and the problem still remain open.

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پوختە

لەم لێکۆڵینەوەیەدا، هەستام بە فراواندنی ئەلگوریسمی[11،12] **بۆ پرۆگرامی کەرتی دووجای. دوو ڕێگام بە کارهێناوە بۆ شیکارکردنی پرۆگرامی کەرتی دووجایی بە بەکارهینانی هەردوو تەکنیکی ڕاستەوخۆی پەسەند و سادەی پەرەسیندراو دەرئە نجام ئە نجامی باش بەدەستهاتن بە بەراورد لەگەڵ هەنگاوەکانی شیکارکردنی هەردوو ڕێگاکە. ئینجا گونجانی خوارزمیە کە لە ڕێگایی شیکارکردنی کێشەکەوە سەلمێنراوە بە بەکارهینانە کومپیوتەریەکان لە شیکارکردنی نموونەیەکی حسابی، بە بەکارهینانی** Matlab 2011 version 7.12.0.635 (R2011a). **کۆتایی لێکۆڵینەوە کە تاییەتکراوە بۆ گفتوگوکردنی ئە نجامەکان .**